Answer all TWENTY SIX questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 A plane flew from Madrid to Dubai.

The distance the plane flew was 5658 km.
The flight time was 8 hours 12 minutes.

Work out the average speed of the plane.

1hr = 5 x 12 mins

8hours = 40 x 12mins

8hr 12mins = 41 x 12mins

5658km = 8hrs 12mins ÷41

138km = 12mins

690 = 60mins x5

690 km/h

(Total for Question 1 is 3 marks)

2 Here are the first 4 terms of an arithmetic sequence.

85 -6 79 -6 73 -6 67

Find an expression, in terms of n, for the nth term of the sequence.

$-6n + 91$

$(91 - 6n)$

$-6n+91$

(Total for Question 2 is 2 marks)

3

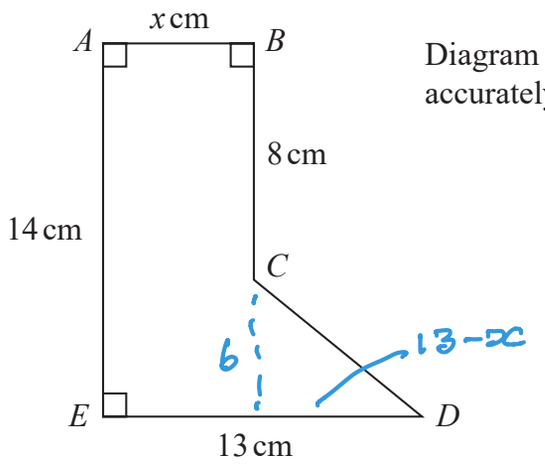

Diagram **NOT** accurately drawn

The diagram shows the shape *ABCDE*.

The area of the shape is 91.8 cm²

Work out the value of *x*.

$91.8 = 14 \times x + \frac{1}{2} \times (13 - x) \times 6^{3}$

$= 14x + 39 - 3x$

$91.8 - 39 = 11x$

$x = \frac{52.8}{11}$

$= 4.8$

$x =$ 4.8

(Total for Question 3 is 4 marks)

4 On a farm there are chickens, ducks and pigs.

The ratio of the number of chickens to the number of ducks is 7 : 2
The ratio of the number of ducks to the number of pigs is 5 : 9
There are 36 pigs on the farm.

Work out the number of chickens on the farm.

C D D : P
7 : 2) x5 x2 (5 : 9
35 : 10 10 : 18

C D P
35 10 18) x2
36

35 x 2 = 70 10 x 2 = 20

70

(Total for Question 4 is 3 marks)

5 (a) Expand and simplify $3x(2x + 3) - x(3x + 5)$

$6x^2 + 9x - 3x^2 - 5x$

$6x^2 - 3x^2 + 9x - 5x$

$3x^2 + 4x$

(2)

(b) Make t the subject of the formula $p = at - d$

$p + d = at$

$t = \frac{p + d}{a}$

$t = \frac{p + d}{a}$

(2)

Given that $\frac{w^5 \times w^n}{w^3} = w^{10}$

(c) work out the value of n.

$(5 + n) - 3 = 10$

$5 + n = 10 + 3$

$5 + n = 13$

$n = 13 - 5 = 8$

$n = 8$

(2)

(Total for Question 5 is 6 marks)

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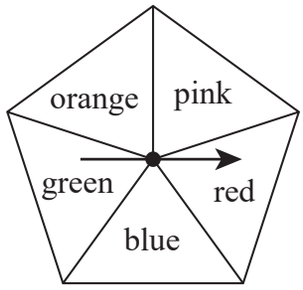
6 Grace has a biased 5-sided spinner.



Grace is going to spin the arrow on the spinner once.

The table below gives the probabilities that the spinner will land on red or on blue or on green.

3x x

Colour	Red	Blue	Green	Orange	Pink
Probability	0.20	0.12	0.08	0.45	0.15

The probability that the spinner will land on orange is 3 times the probability that the spinner will land on pink.

(a) Work out the probability that the spinner will land on orange.

$1 - (0.2 + 0.12 + 0.08)$

$= 1 - 0.4 = 0.6$

$0.6 \div 4 = 0.15$

$0.15 \times 3 = 0.45$

0.45

(3)

Grace spins the arrow on the spinner 150 times.

(b) Work out an estimate for the number of times the spinner lands on blue. 0.12

0.12×150

18

(2)

(Total for Question 6 is 5 marks)

7 $-4 \leqslant 2y < 6$

y is an integer.

(a) Write down all the possible values of y.

$-4 \leqslant 2y < 6$

$\div 2 \quad \div 2 \quad \div 2$

$-2 \leqslant y < 3$

-2 -1 0 1 2

(2)

(b) Solve the inequality $7t - 3 \leqslant 2t + 31$

Show your working clearly.

$7t - 3 \leqslant 2t + 31$

$-2t \qquad -2t$

$5t - 3 \leqslant 31$

$+3 \qquad +3$

$5t \leqslant 34$

$t \leqslant \frac{34}{5}$

$t \leqslant 6.8$

(2)

(Total for Question 7 is 4 marks)

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8 The table shows the populations of five countries.

Country	Population
China	1.4×10^9
Germany	8.2×10^7
Sweden	9.9×10^6
Fiji	9.1×10^5
Malta	4.3×10^5

(a) Work out the difference between the population of China and the population of Germany.
Give your answer in standard form.

$1.4 \times 10^9 - 8.2 \times 10^7$

1318000000

1.318×10^9

(2)

Given that

$$\text{population of Fiji} = \frac{1}{k} \times \text{population of Sweden}$$

(b) work out the value of k.
Give your answer correct to the nearest whole number.

$9.1 \times 10^5 = \frac{1}{k} \times 9.9 \times 10^6$

$k = \frac{9.9 \times 10^6}{9.1 \times 10^5}$

$= 10.879...$

11

(whole number)

$k =$ 11

(2)

(Total for Question 8 is 4 marks)

9 (a) Factorise fully $25a^4c^7d + 45a^9c^3h$

$5a^4c^3(5c^4d + 9a^5h)$

(2)

(b) Solve $(2x + 5)^2 = (2x + 3)(2x - 1)$

$4x^2 + 20x + 25 = 4x^2 + 4x - 3$

$16x = -28$

$x = -\frac{28}{16}$

$= -\frac{7}{4}$

$x = -1.75$

(3)

(Total for Question 9 is 5 marks)

10 Jethro has sat 5 tests.
Each test was marked out of 100 and Jethro's mean mark for the 5 tests is 74

Jethro has to sit one more test that is also to be marked out of 100

Jethro wants his mean mark for all 6 tests to be at least 77

Work out the least mark that Jethro needs to get for the last test.

5 tests. mean = 74 Total score = $5 \times 74 = 370$

6 tests mean = 77 Total score = $6 \times 77 = 462$

$462 - 370 = 92$

92

(Total for Question 10 is 3 marks)

11 $\sqrt{2} \times 16 = 2^x$

$\sqrt{2} = 2^{1/2}$ $16 = 2^4$

(a) Find the value of x.
Show your working clearly.

$2^{\frac{1}{2}} \times 2^4 = 2^{4.5}$

$x =$ 4.5

(2)

$$\frac{(11^{-6})^5}{11^4} = 11^n$$

(b) Find the value of n.
Show your working clearly.

$11^{-6 \times 5} = 11^{-30}$

$\frac{11^{-30}}{11^4} = 11^{-30-4} = 11^{-34}$

$n =$ 11^{-34}

(2)

(Total for Question 11 is 4 marks)

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12 The diagram shows a sector of a circle with radius 7 cm.

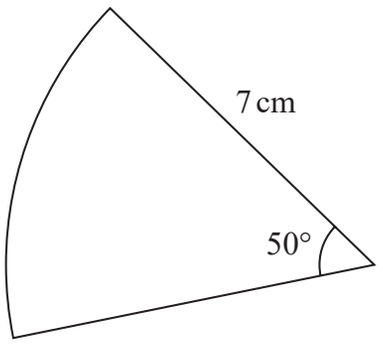

Diagram **NOT** accurately drawn

Work out the length of the arc of the sector.
Give your answer correct to one decimal place.

$r = 7$ $d = 14$ $C = \pi d$ $A = \pi r^2$

$\frac{50}{360} \times \pi \times 14$

$= 6.10865$

(1 dp)

6.1 ..... cm

(Total for Question 12 is 2 marks)

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13 Expand and simplify $4x(3x + 1)(2x - 3)$
Show your working clearly.

$4x(6x^2 - 9x + 2x - 3)$

$= 4x(6x^2 - 7x - 3)$

$= 24x^3 - 28x^2 - 12x$

$24x^3 - 28x^2 - 12x$

(Total for Question 13 is 3 marks)

14 Here is the number of goals that Henri's team scored one summer in each water polo match.

5 8 9 11 13 13 14 15 16 17 20

LQ UQ

Find the interquartile range of the numbers of goals.
Show your working clearly.

$16 - 9 = 7$

7

(Total for Question 14 is 2 marks)

15 P, Q and R are points on a circle, centre O.
TRV is the tangent to the circle at R.

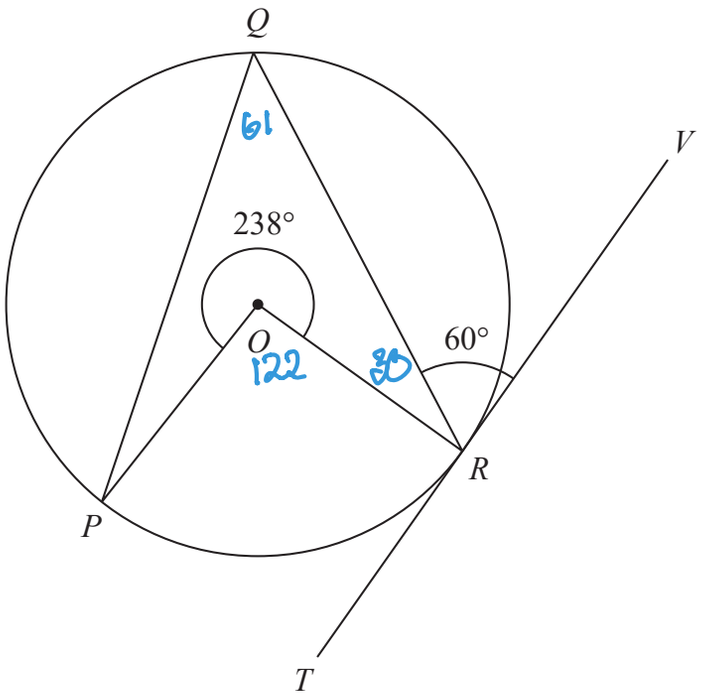


Diagram **NOT** accurately drawn

Reflex angle $POR = 238°$
Angle $QRV = 60°$

Calculate the size of angle OPQ.
Give a reason for each stage of your working.

POR = 360 - 238 = 122 angles around a point = 360°

PQR = $\frac{122}{2}$ = 61 angle at the centre is twice the angle at the circumference

ORQ = 90 - 60 = 30 angle between tangent and radius = 90°

OPQ = 360 - (238 + 30 + 61)
= 31 angles in a quadrilateral = 360

31 °

(Total for Question 15 is 4 marks)

16 Use algebra to show that the recurring decimal $0.28\dot{1}\dot{3} = \frac{557}{1980}$

$100x = 28.131313...$

$x = 0.281313...$

$99x = 27.85$

$x = \frac{27.85}{99}$ $\quad \frac{27.85}{99} = \frac{2785}{9900}$ $\quad \frac{2785}{9900} \overset{\div 5}{\underset{\div 5}{=}} \frac{557}{1980}$

as required

(Total for Question 16 is 2 marks)

17 Using algebra, prove that, given any 3 consecutive even numbers, the difference between the square of the largest number and the square of the smallest number is always 8 times the middle number.

$2n$ = first even number

$2n+2$, $2n+4$ } consecutive even numbers

$(2n+4)^2 - (2n)^2 = 4n^2 + 16n + 16 - 4n^2 = 16n + 16$

$8 \times (2n+2) = 16n + 16$

$\therefore \quad 16n + 16 \equiv 16n + 16$

(Total for Question 17 is 3 marks)

18 The table and histogram give information about the distance travelled, in order to get to work, by each person working in a large store.

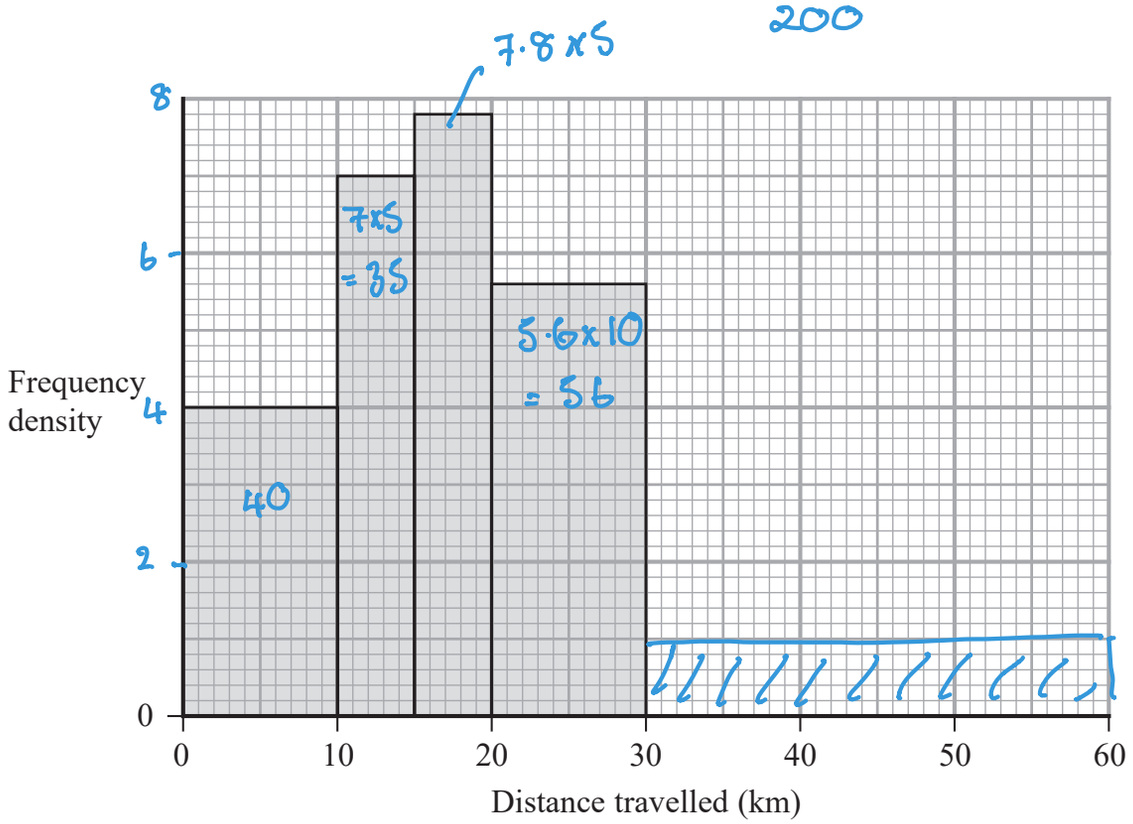
F.D

40 ÷ 10 = 4

Distance (d km)	Frequency
$0 \leqslant d < 10$	40
$10 \leqslant d < 15$	35
$15 \leqslant d < 20$	39
$20 \leqslant d < 30$	56
$30 \leqslant d < 60$	30

30 ÷ 30 = 1

200



part c) > 25

5.6 x 5 = 28 + 30

= 58

Using the information in the table and in the histogram,

(a) complete the table,

✓

(2)

(b) complete the histogram.

✓

(1)

One of the people working in the store is chosen at random.

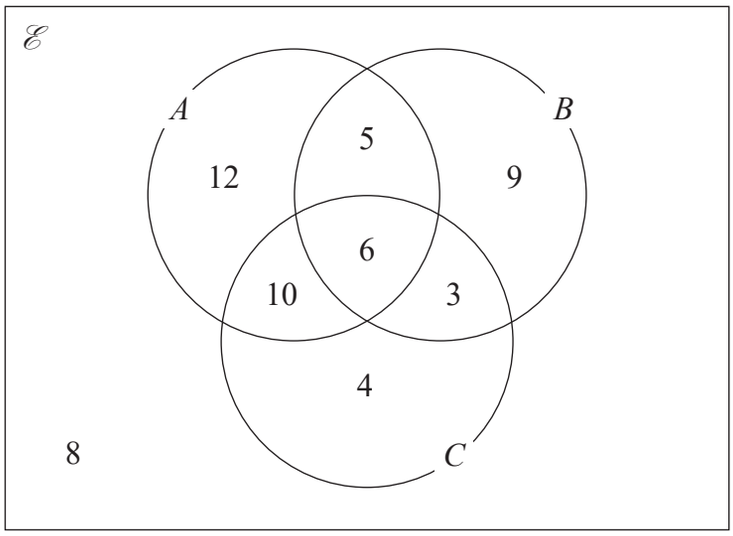
(c) Work out an estimate for the probability that the distance travelled by this person, in order to get to work, was greater than 25 km.

$\frac{58}{200}$

(2)

(Total for Question 18 is 5 marks)

19 The Venn diagram shows a universal set, $\mathscr{E}$ and sets A, B and C.



12, 5, 9, 10, 6, 3, 4 and 8 represent the **numbers** of elements.

Find

(i) $n(A \cup B)$

12 + 10 + 6 + 5 + 3 + 9

45

(1)

(ii) $n(A' \cap B')$

8 + 4

12

(1)

(iii) $n([A \cap B] \cup C)$

5 + 6 + 10 + 4 + 3

28

(1)

(Total for Question 19 is 3 marks)

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20 $P = \dfrac{t - w}{y}$

t = 9.7 correct to 1 decimal place

w = 5.9 correct to 1 decimal place

y = 3 correct to 1 significant figure

Calculate the upper bound for the value of P.
Show your working clearly.

9.7 → 9.75, 9.65

5.9 → 5.95, 5.85

2 → 2.5, 3 → 3.5, 4

$P_{UB} = \dfrac{9.75 - 5.85}{2.5}$

$= 1.56$

1.56

(Total for Question 20 is 3 marks)

21 Given that $x = \dfrac{5}{9y + 5}$ and that $y = \dfrac{5}{5a - 2}$

find an expression for x in terms of a.
Give your expression as a single fraction in its simplest form.

$x = 5 \div (9y + 5)$

$= 5 \div 9\left(\dfrac{5}{5a-2}\right) + 5$

$= 5 \div \dfrac{45}{5a-2} + 5$

$= 5 \div \dfrac{45 + 5(5a-2)}{5a-2}$

$= 5 \div \dfrac{45 + 25a - 10}{5a-2} = 5 \div \dfrac{35 + 25a}{5a-2}$

$= 5 \times \dfrac{5a-2}{35+25a} = \dfrac{25a-10}{35+25a}$

$x = \dfrac{25a-10}{35+25a} \div 5 = \dfrac{5a-2}{7+5a}$

$\dfrac{5a-2}{7+5a}$

(Total for Question 21 is 4 marks)

22 The diagram shows a triangular prism *ABCDEF* with a horizontal base *ABEF*.

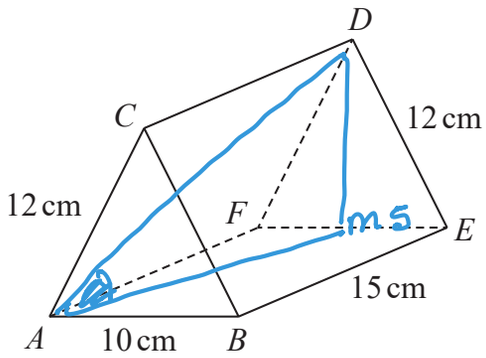

Diagram **NOT** accurately drawn

$AC = BC = FD = ED = 12\,\text{cm}$ $AB = 10\,\text{cm}$ $BE = 15\,\text{cm}$

Calculate the size of the angle between *AD* and the base *ABEF*.
Give your answer correct to 3 significant figures.

let M be midpoint of FE

DM => $\sqrt{12^2 - 5^2} = \sqrt{144-25} = \sqrt{119}$

AM => $\sqrt{15^2+5^2} = \sqrt{250}$

angle $\tan x = \frac{\sqrt{119}}{\sqrt{250}}$

$x = \tan^{-1} \frac{\sqrt{119}}{\sqrt{250}}$

$= 34.602...$

(3sf.)

34.6 °

(Total for Question 22 is 4 marks)

23 The sum of the first N terms of an arithmetic series, S, is 292
The 2nd term of S is 8.5
The 5th term of S is 13

Find the value of N.
Show clear algebraic working.

$a + d = 8.5$

$a + 4d = 13$

$3d = 4.5$

$d = 1.5$ $\quad a + 1.5 = 8.5$

$\therefore a = 7$

$S_n = \frac{N}{2}(2a + (N-1)d)$

$= \frac{N}{2}(2 \times 7 + (N-1)1.5) = 292$

$\times 2 \Rightarrow N(14 + 1.5N - 1.5) = 584$

$\Rightarrow 12.5N + 1.5N^2 - 584 = 0$

$\times 2$

$= 3N^2 + 25N - 1168 = 0$

$(3N + 73)(N - 16) = 0$

$-\frac{73}{3}$ $\quad N = 16$

not valid $\quad \therefore N = 16$

$N =$ 16

(Total for Question 23 is 5 marks)

24 The functions f and g are defined as

$$f(x) = 5x^2 - 10x + 7 \qquad \text{where } x \geqslant 1$$

$$g(x) = 7x - 6$$

(a) Find fg(2)

$g(2) = 7 \times 2 - 6 = 8$

$fg(2) = 5 \times (8)^2 - 10 \times 8 + 7$

$=$

.....

(2)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$y = 5(x^2 - 2x) + 7$

$= 5[(x-1)^2 - 1] + 7$

$= 5(x-1)^2 - 5 + 7$

$= 5(x-1)^2 + 2$

$\frac{y-2}{5} = (x-1)^2$

$x - 1 = \sqrt{\frac{y-2}{5}}$

$x = 1 + \sqrt{\frac{y-2}{5}}$

$f^{-1}(x) = 1 + \sqrt{\frac{y-2}{5}}$

(4)

(Total for Question 24 is 6 marks)

25 The diagram shows two circles such that the region **R**, shown shaded in the diagram, is the region common to both circles.

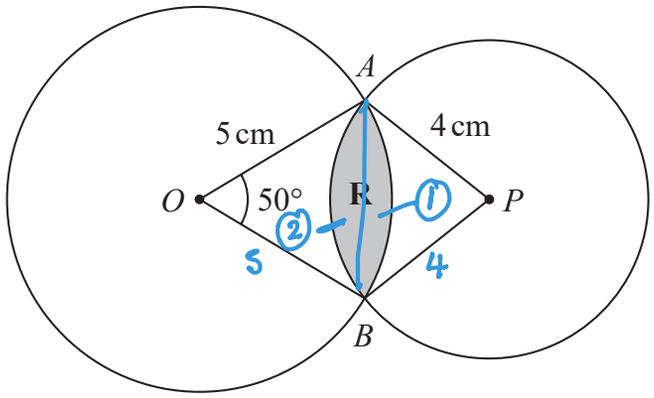

Diagram **NOT** accurately drawn

One of the circles has centre O and radius 5 cm.
The other circle has centre P and radius 4 cm.
Angle $AOB = 50°$

Calculate the area of region **R**.
Give your answer correct to 3 significant figures.

Chord AB $\quad AB^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 50$

$AB = 4.226182617$

angle APB $\quad \cos A = \dfrac{4^2 + 4^2 - 4.226..^2}{2 \times 4 \times 4}$

$A = \cos^{-1} 0.441$

$A = 63.7776..$

① sector OAB

$\frac{50}{360} \times \pi \times 5^2 = 10.9083...$

$\triangle OAB = \frac{1}{2} \times 5 \times 5 \times \sin 50 = 9.57555...$

$10.9083... - 9.57555...$

area ① $= 1.3327...$

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DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

② sector ABP

$\frac{63.776..}{360} \times \pi \times 4^2$ = 8.9058...

$\triangle ABP = \frac{1}{2} \times 4 \times 4 \times \sin 63.7776$ = 7.17668...

area 2 = 1.728..

Shaded area

= 1.3327.. + 1.728

= 3.06...

3.06 cm^2

(Total for Question 25 is 6 marks)

Turn over for Question 26

26 *OACB* is a trapezium.

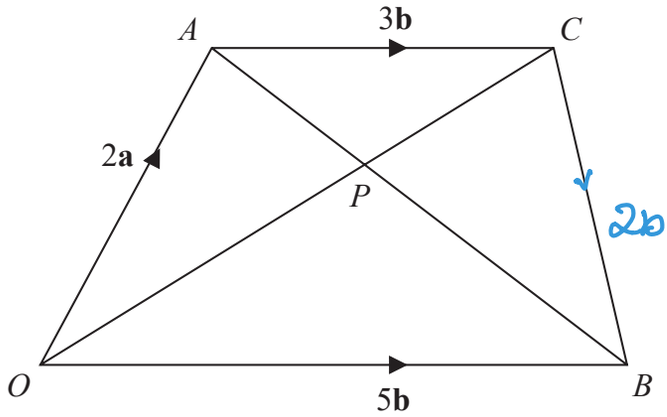

Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{2a}$ $\overrightarrow{OB} = \mathbf{5b}$ $\overrightarrow{AC} = \mathbf{3b}$

The diagonals, *OC* and *AB*, of the trapezium intersect at the point *P*.

Find and simplify an expression, in terms of **a** and **b**, for $\overrightarrow{OP}$
Show your working clearly.

$\overrightarrow{CB} = -3b - 2a + 5b = 2b - 5a$

$\overrightarrow{OC} = 2a + 3b \rightarrow OP = m(2a + 3b)$

$\overrightarrow{AB} = 5b - 2a$

$OP = 2a + n(5b - 2a)$

$OP = 2ma + 3mb$ and $OP = 2a + 5nb - 2na$

$= (2 - 2n)a + 5nb$

$\therefore 2m = 2 - 2n$

$3m = 5n$

$m = \frac{5}{3}n$

$2 \times \frac{5}{3}n = 2 - 2n$

$\frac{10}{3}n + 2n = 2$

$\frac{16}{3}n = 2$

$n = \frac{6}{16} = \frac{3}{8}$

$m = \frac{5}{3} \times \frac{3}{8}$

$= \frac{5}{8}$

Sub into $OP = 2ma + 3mb$

$= 2 \times \frac{5}{8} a + 3 \times \frac{5}{8} b$

$= \frac{10}{8} a + \frac{15}{8} b$

$= \frac{5}{4} a + \frac{15}{8} b$

$\overrightarrow{OP} = \frac{5}{4} a + \frac{15}{8} b$

(Total for Question 26 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS